EE178 First Recitation
Electric Charges & Electric Fields

Part I: Electric charges

Exercise 1 :

Three particles having charges of $Q_1 = 4\ \mu C$, $Q_2 = -8\ \mu C$ and $Q_3 = -6\ \mu C$ were placed on the corners of an equilateral triangle with edge equal to L= 1.2 m (See the figure below).

1. Determine the net magnitude and direction of the electrical force applied on charge Q_1 from charges Q_2 and Q_3.
2. Determine the net magnitude and direction of the electrical force applied on charge Q_2 from charges Q_1 and Q_3.
3. Determine the net magnitude and direction of the electrical force applied on charge Q_3 from charges Q_1 and Q_2.

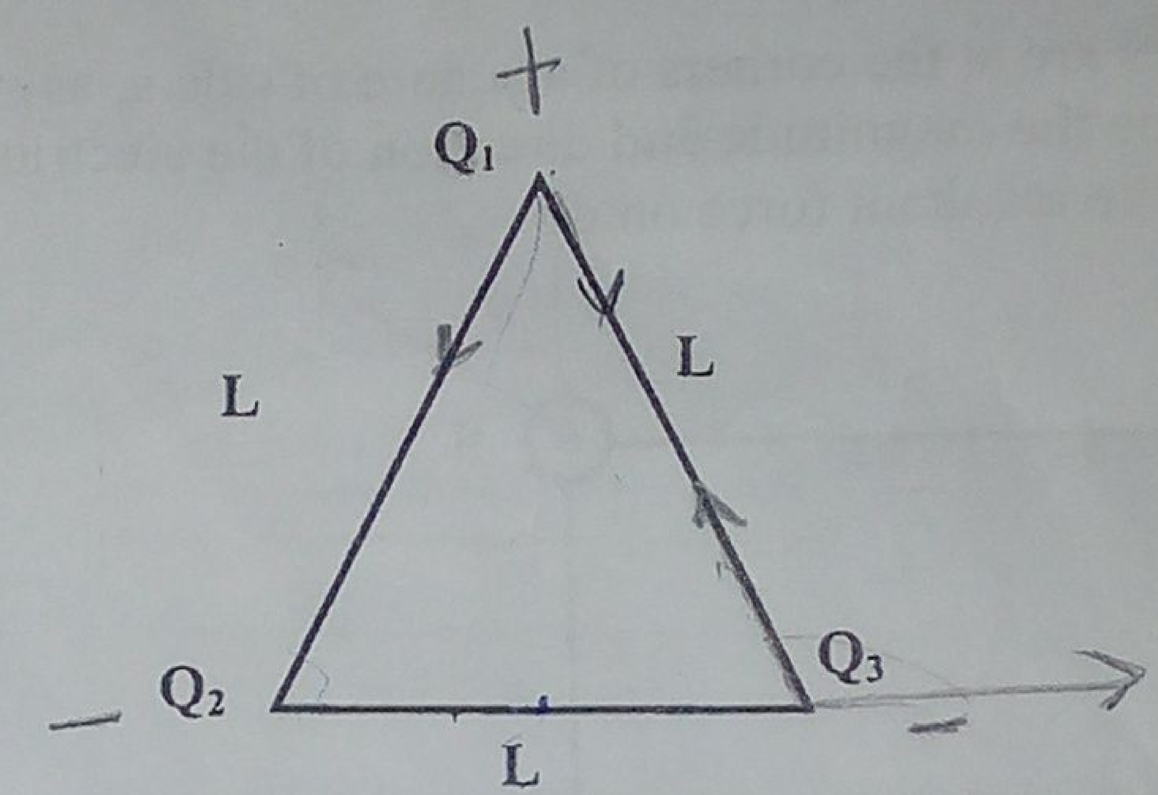

Figure 1

Exercise 2:

Two small beads having positive charges 3q and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point x = d. As shown in the figure 2, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?

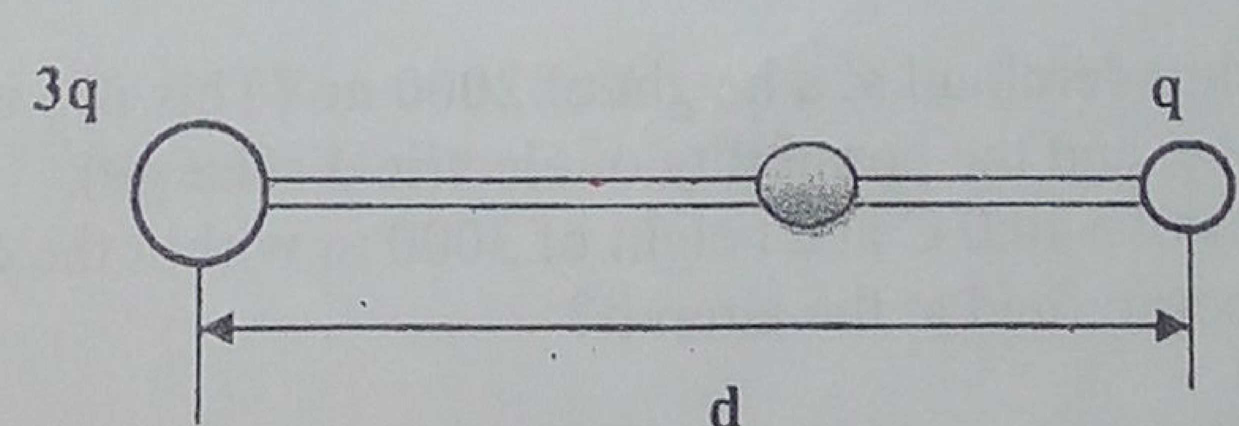

Figure 2

Exercise 3:

Two identical conducting small spheres are placed with their centers 0.300 m apart. One is given a charge of 12.0 nC, and the other is given a charge of -18.0 nC.

1. Find the electric force exerted on one sphere by the other.

2. The spheres are connected by a conducting wire. Find the electric force between the two after equilibrium has occurred.

Exercice 4:

In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is 0.529×10^{-10} m. $m = 9.11 . 10^{-31}$

1. Find the electric force between the two
2. If this force causes the centripetal acceleration of the electron, what is the speed of the electron?

Part II: Electric fields

Exercise #5:

Four point charges are at the corners of a square of side a, as shown in figure 2.

1. Determine the magnitude and direction of the electric field at the location of charge q
2. What is the resultant force on q?

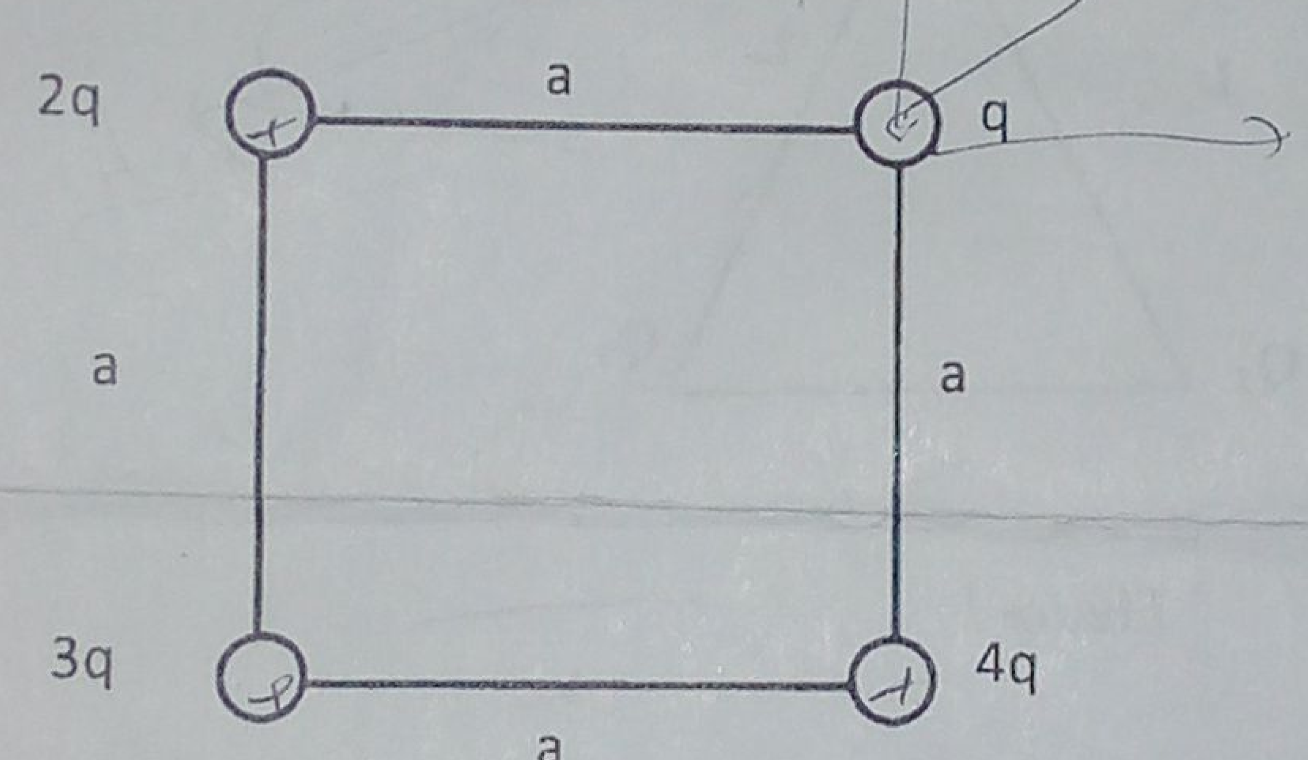

Figure 3

Exercise 6:

An airplane is flying through a thundercloud at a height of 2000 m. (This is a very dangerous thing to do because of updrafts, turbulence, and the possibility of electric discharge).
If there are charge concentrations of +40.0 C at a height of 3000 m within the cloud and of -40.0 C at height of 1000 m, what is the electric field at the aircraft?

Exercise 7:

In a certain thundercloud charges of +45 C and -45 C build up, separated by 6 km. What is the dipole moment of these charges?

Exercise 8:

The HCl molecule has a dipole moment of 3.4×10^{-30} C.m. What torque does the molecule experience in an electric field of 4×10^{6} V/m when the axis of the dipole makes an angle of 30° with the direction of the electric field.

Part III: Continuous charges

Exercise 9:

A thin rod of length L and uniform charge per unit length λ lies along the x-axis, as shown in figure 3.

1. Show that the electric field at P, a distance y from the rod, along the perpendicular bisector (منتصف) has no x component and is given by $E = 2\, k_e\, \lambda\, \sin\theta_0 / y$.
2. Using your result to part 1. , show that the field of a rod of infinite length is: $E = 2\, k_e\, \lambda / y$.
(Hint: First calculate the field at P due to an element of length dx, which has a charge of λdx. Then change variables from x to θ using the fact that $x = y \tan\theta$ and integrate over θ).

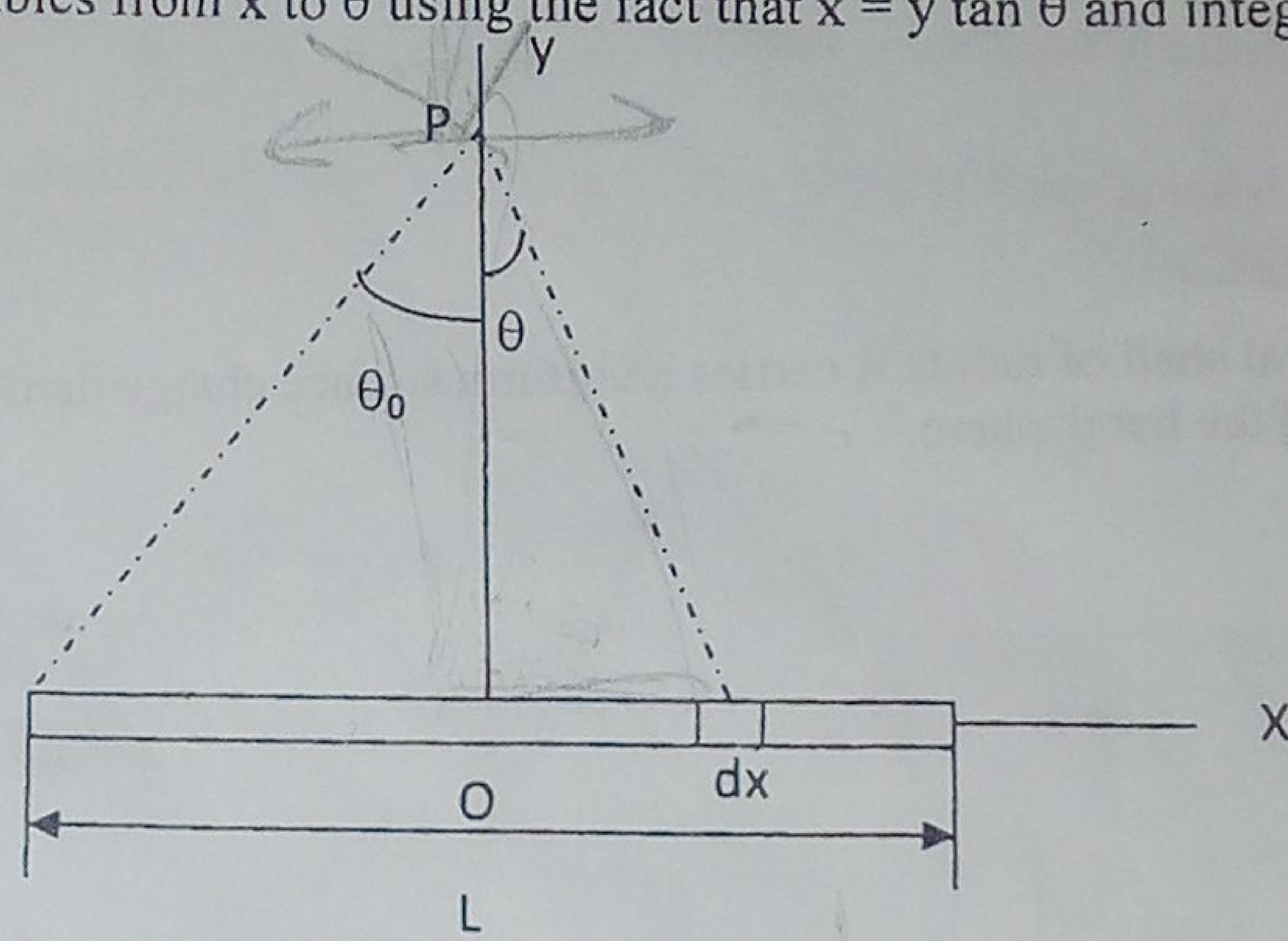

Figure 4

Exercise #10:

Charge Q is distributed uniformly along a rod. The rod is then bent to form a semicircle of radius R. What is the electric field at the center of the semicircle?

Exercise #11:

A line of charge starts at $x = +x_0$ and extends to positive infinity. If the linear charge density is $\lambda = \lambda_0\, x_0/x$, determine the electric field at the origin.

Exercise #12:

A uniformly charged disk of radius 35.0 cm carries a charge density of 7.90×10^{-3} C/m^2. Calculate the electric field on the axis of the disk at 5.00 cm from the center of the disk.

Exercise 13:

Identical thin rods of length 2a carry equal charges, +Q, uniformly distributed along their lengths. The rods lie along the x axis with their centers separated by a distance of b > 2a (Figure 5). Show that the magnitude of the force exerted by the left rod on the right one is given by:

$$F = \left(\frac{k_e Q^2}{4a^2}\right) \ln\left(\frac{b^2}{b^2 - 4a^2}\right)$$

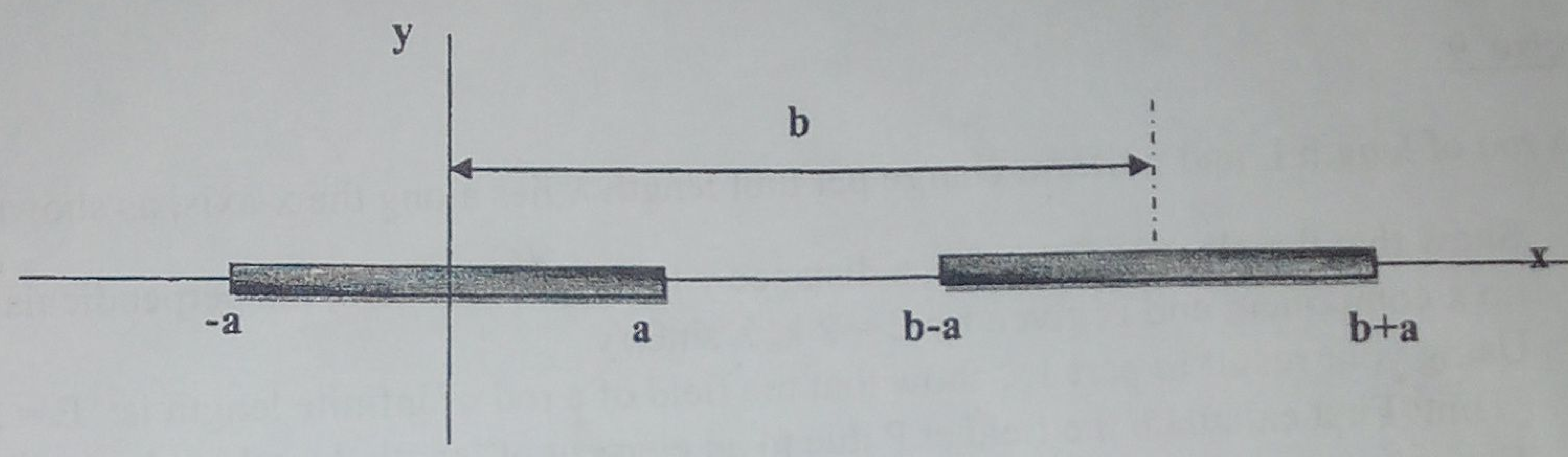

Figure 5

Exercise 14:

A hemispherical shell of radius R carries a uniform surface charge density. Determine the electric field at the center of the hemisphere.

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EE178 Second Recitation
Gauss' Law

Exercise 1 :

An insulating sphere of radius **R** has a charge density $\rho = a \times r$ where **a** is a constant. Let **r** be the distance from the center of the sphere. Find the electric field everywhere, both inside and outside the sphere.

Exercise 2:

1. Compute the electric field through a square surface of edges **2l** due to a charge **+Q** located at a perpendicular distance **l** from the center of the square, as shown in the figure below.

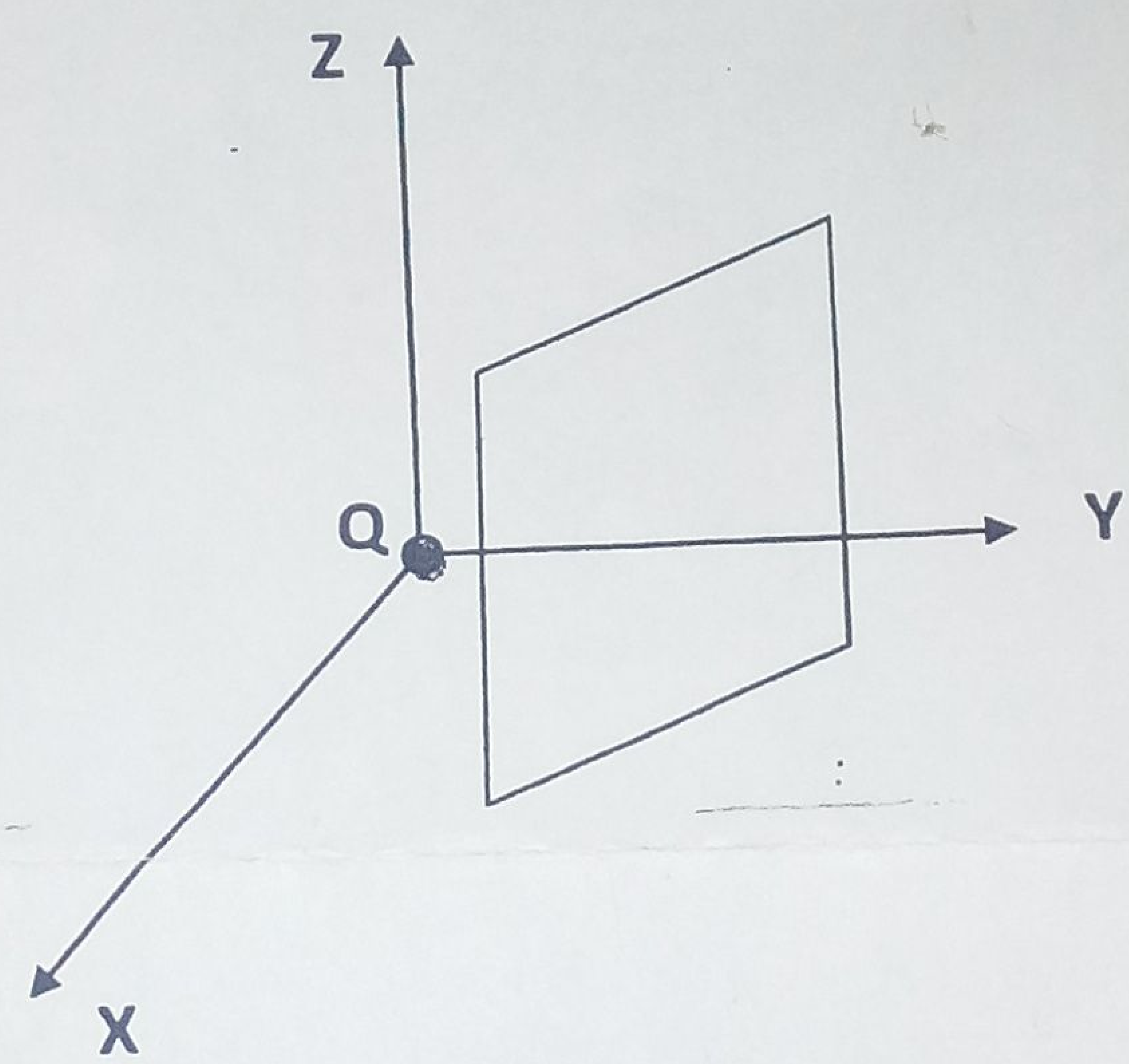


Figure 1

2. Using the result obtained previously, if the charge +Q is now at the center of a cube of side **2×l** , what is the total flux emerging from all the six faces of the closed surface.

Exercise 3:

Find the electric field inside and outside a thin walled tube of radius R = 3.0 cm and carrying a positive charge per unit length $\lambda = 2.0\times10^{-8}$ C/m

Exercice 4:

Three concentric hollow metallic spherical shell of radii a, b, and c carry charges +2Q, -3Q, and +Q, respectively. Determine the charge on the inner surface and on the outer surface of each sphere.

Exercise 5:

A solid insulating sphere of radius **a** has a uniform charge density ρ and a total charge **Q.**

Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are **b** and **c** as shown in the figure below:

1. Find the magnitude of the electric field in the following regions: $r < a$; $a<r<b$; $b<r<c$; $r>c$
2. Determine the induced charge per unit area on the inner and outer surfaces of the hollow spheres.

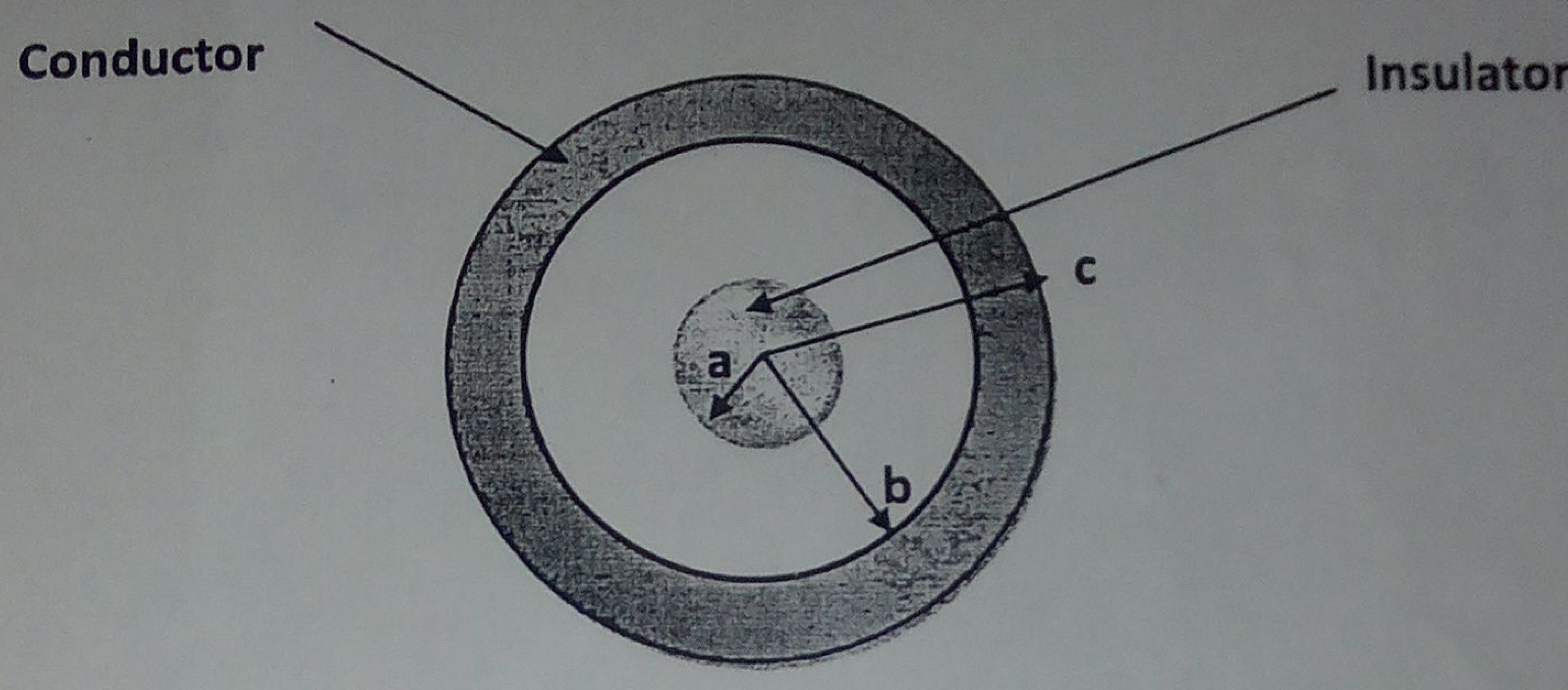


Figure 2

Exercise 6:

Charge is distributed with constant density ρ throughout a sphere of radius **R**. There is a spherical void of radius **R/2** within the large sphere, positioned as shown in the figure. Show that the electric field within the void is constant in magnitude and direction.

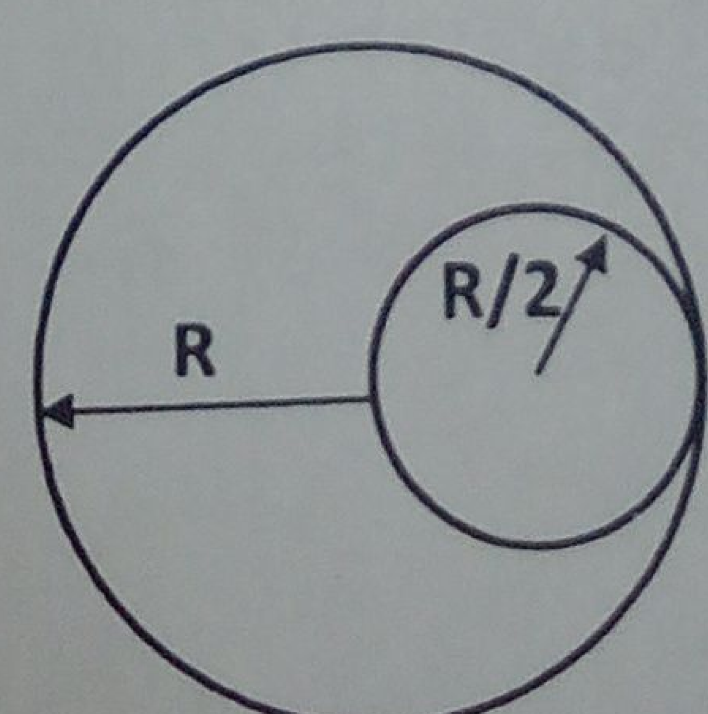


Figure 3

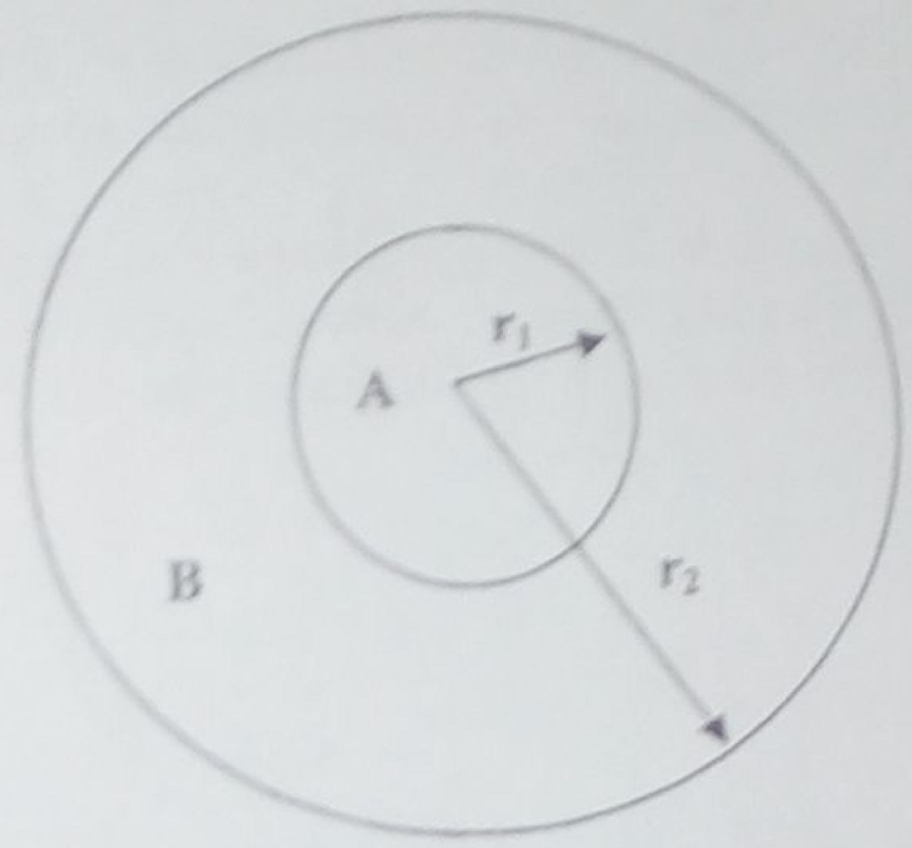
R-2

EE178 Third Recitation
Electric potential

Exercise 1:

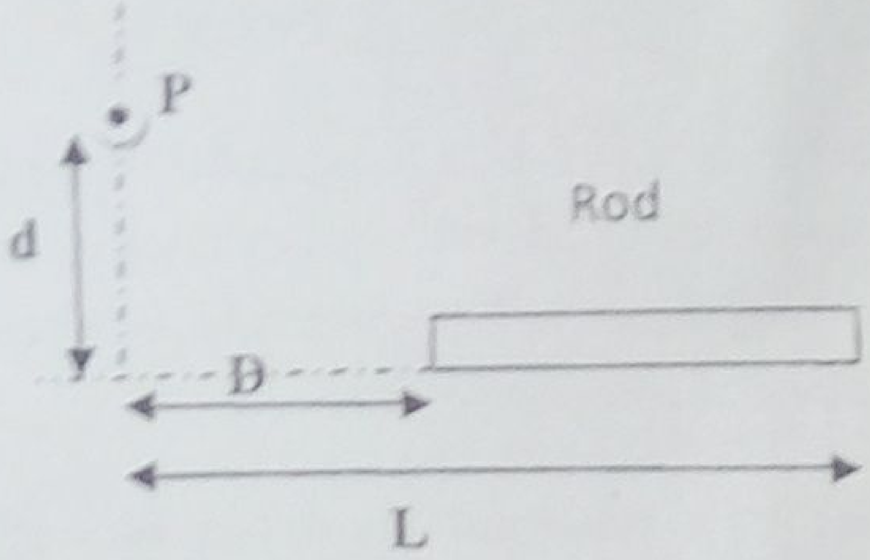
Consider two thin, conducting, spherical shells as shown in the figure below. The inner shell has a radius r_1 = 15.0 cm and a charge 10.0 nC. The outer shell has a radius r_2 = 30.0 cm and a charge -15.0 nC. Find:

1. The electric field $\vec{E}$
2. The electric potential V in regions A, B and C, with V = 0 at r = ∞



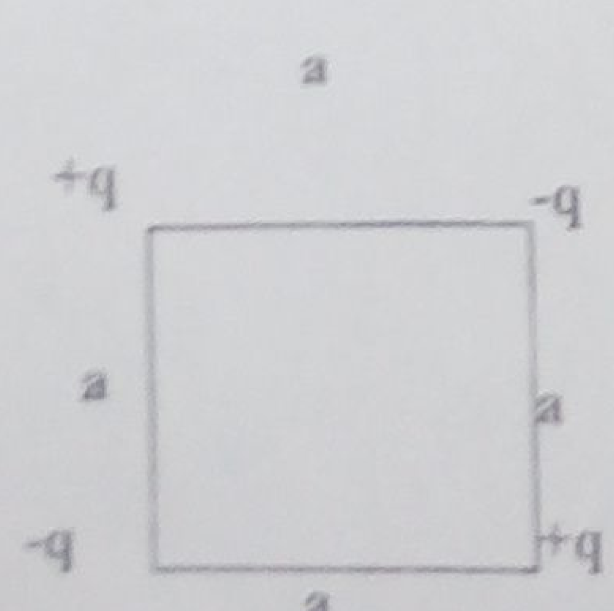
Exercise 2:

The figure below shows a thin rod with a uniform charge density of 1.00 μC/m. Evaluate the electric potential at P if d = D = L/4



Exercise 3:

How much work is required to set up the arrangement of the figure if q = 2.30 pC, a = 64.0 cm and the particles are infinitely far apart and at rest?



Exercice 4:

A total charge of 5.4×10^{-6} C is uniformly distributed along a ring of radius 0.89 m.

1. What is the potential at the center of the ring?
2. What is the potential at a point on the axis of the ring at a distance of 0.98 m from the plane of the ring?

Exercise 5:

Two parallel plates carry a surface charge density of $+\sigma$ and $-\sigma$ respectively, and are separated by a small distance d. Assume that the size of the plates is always large compared with the distance to the plates.

1. What is the electric field in the region between the plates?
2. What is the potential difference between a point on one plate and a point on the other plate?
3. Which plate, the positive or the negative plate, is at the higher potential?

Exercise 6:

What is the magnitude of the electric field at the point $(3\vec{\imath} - 2\vec{\jmath} + 4\vec{k})m$ if the electric potential is given by $V = 2xyz^2$, where V is in volts and x, y and z are in meters?

Exercise 7:

A coaxial cable consists of a long, conducting wire, of radius $\mathbf{R_1}$ with a linear charge density of λ, and a long conducting coaxial spherical cylindrical shell, with an inner radius $\mathbf{R_2}$ and an outer radius R_3, and with a symmetric linear density $-\lambda$. We assume the length to be much greater than any of the radial distances of interest.

1. What is the potential due to the cable at a point at a radial distance from the axis r, such that $r > R_3$
2. What is the potential at a point within the outer cylindrical shell at $R_2 < r < R_3$?
3. What is the potential at a point between the wire and the cylinder at $R_1 < r < R_2$?
4. What is the potential at a point within the wire $r < R_1$?

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Table of Basic Integrals

(1) $$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1$$

(2) $$\int \frac{1}{x}\,dx = \ln|x|$$

(3) $$\int u\,dv = uv - \int v\,du$$

(4) $$\int e^x\,dx = e^x$$

(5) $$\int a^x\,dx = \frac{1}{\ln a}a^x$$

(6) $$\int \ln x\,dx = x\ln x - x$$

(7) $$\int \sin x\,dx = -\cos x$$

(8) $$\int \cos x\,dx = \sin x$$

(9) $$\int \tan x\,dx = \ln|\sec x|$$

(10) $$\int \sec x\,dx = \ln|\sec x + \tan x|$$

(11) $$\int \sec^2 x\,dx = \tan x$$

(12) $$\int \sec x\tan x\,dx = \sec x$$

(13) $$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\frac{x}{a}$$

(14) $$\int \frac{a}{a^2-x^2}\,dx = \frac{1}{2}\ln\left|\frac{x+a}{x-a}\right|$$

(15) $$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}$$

(16) $$\int \frac{a}{x\sqrt{x^2-a^2}}\,dx = \sec^{-1}\frac{x}{a}$$

(17) $$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\frac{x}{a} = \ln(x+\sqrt{x^2-a^2})$$

(18) $$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\frac{x}{a} = \ln(x+\sqrt{x^2+a^2})$$